Suppression of chaos by nonresonant parametric perturbations

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It is shown analytically and numerically that the suppression of chaos may be effectively achieved by applying a high-frequency parametric force to a chaotic dynamical system. Such a periodic nonresonant force may decrease or even completely eliminate chaos. Taking the Duffing oscillator as a concrete but rather general example, an analytical approach is elaborated to demonstrate how such a suppression of chaos may be understood in the framework of the effective "averaged" nonlinear equation for a slowly varying component of the oscillation amplitude. As follows from our numerical simulations, the suppression of chaos may be observed not only at large amplitudes of the parametric force but also at smaller amplitudes, showing a decay of the leading Lyapunov exponent within certain amplitude-frequency "windows."

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I. INTRODUCTION

Dynamical chaos is a very interesting nonlinear phenomenon, and it has been detected in a large number of nonlinear systems of various physical nature. In practice, however, this effect is usually undesirable, because it restricts the operating range of many electronic and mechanic devices. One of the main methods to control or completely eliminate chaotic dynamics is based on the idea of the stabilization of unstable periodic orbits embedded within a strange attractor. This may be achieved by making a *small* time-dependent perturbation in the form of feedback to an accessible system parameter (see, e.g., [1–5]). Another way is to apply an external force (see, e.g., Refs. [6–10]). The use of a resonant response of chaotic systems to continuous *external* periodic perturbations to suppress chaos in dynamical systems has been proposed in [6, 7], and then considered numerically [7], analytically [7, 8], and even experimentally [9]. In particular, as shown in [7], a small *parametric* perturbation of the nonlinear Duffing oscillator [11] showing chaotic dynamics may reduce or even completely suppress chaos. The basic idea proposed in [7, 9] is to apply a parametric force at some *resonant* frequency (related to the frequency of the primary chaos-inducing periodic force). "Laminar" phases in the system dynamics are then observed of increasing duration up to complete regularization of the motion at *exact resonance*.

The results displayed in [7, 9] do show an efficient suppression of chaos at resonant conditions, the most effective suppression being observed at the main resonance. It is clear, however, that such a condition is rather special and, strictly speaking, two external forces of different origin applied to a dynamical system are of incommensurate frequencies. The combined effect of two periodic (external and parametric) forces with different frequencies of the same order has recently been analyzed [12] (see also [13]) by using the Melnikov method [14, 15] showing a rather complicated set of bifurcations. However, it is well known that a parametric force of a *high frequency* (HF) may drastically change the dynamics of the phase trajectories of an averaged nonlinear system, and this effect may lead, e.g., to a *stabilization* of certain types of dynamical regimes. A typical and famous example is the stabilization of a reverse pendulum by parametrically forced oscillations of its pivot [16], and the similar effect may also be achieved by a direct force of large amplitude [17]. Dynamical stabilization has its analog in nonlinear systems with distributed parameters supporting, in particular, novel types of kink solitons [18–21].

The purpose of the present paper is to show analytically and numerically that the suppression of chaos may be achieved efficiently by applying a *nonresonant* parametric force of high frequency. The main idea of the method we propose is based on the observation that parametric perturbations can change the stability properties of elliptic or hyperbolic points on the phase plane of an *averaged* dynamical system. This property is rather general, and the parametrically induced suppression of chaos, therefore, may be achieved in dynamical systems of various physical origin. The method itself does not require any real-time measurement of the system dynamics nor any real-time calculations and this, as we hope, will

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allow to control fast systems (e.g., faster ones than accessible for the method proposed by Ott, Grebogi, and Yorke [1]). For presenting the basic ideas of the method, we take the well-known Duffing oscillator driven by an external (direct) periodic force which produces chaos and apply a HF parametric force of arbitrary amplitude.

The paper is organized as follows. In Sec. II we present the model and, by using an asymptotic expansion to split slow and fast oscillations generated by the parametric force, we derive an effective ("averaged") Duffing equation for the slowly varying oscillation component. This averaged equation shows how chaotic dynamics may be suppressed. We discuss such a suppression in Sec. III by means of the Melnikov function and by numerical simulations. Finally, conclusions are drawn in Sec. IV.

II. "AVERAGED" EQUATION

Let us consider the driven and damped Duffing oscillator with a parametric force

$$\ddot{x} - \alpha(t)x + \beta x^3 = -\gamma \dot{x} + F\cos(\omega t), \tag{1}$$

where, for simplicity,

$$\alpha(t) = \alpha[1 + \epsilon \cos(\Omega t)], \tag{2}$$

Ω being the frequency of the parametric force which is assumed to be large in comparison with the direct driving frequency ω.

Considering the parametric force as rapidly oscillating, we apply an analytical method based on separation of different time scales. The basic idea to split fast and slow variables is not new, and the well-known example is a stabilization of the reverse pendulum by oscillations of its suspension point (see [16]). However, our analytical method to derive an effective equation for a slowly varying oscillation component allows us to get all the corrections in a self-consistent way by using asymptotic expansions (see also [22] where a similar approach has been recently applied to a soliton problem).

In order to derive the equation of motion for the slowly varying dynamics, we *decompose* the function $x(t)$ into a sum of slowly and rapidly varying parts, i.e.,

$$x = X + \xi. \tag{3}$$

The function $\xi(t)$ stands for fast oscillations around the slowly varying envelope function $X(t)$, and the mean value of $\xi(t)$ during an oscillation period is assumed to be zero so that $\langle x \rangle = X$. Our aim is to derive an effective equation for the function X. The rapidly varying parametric force generates the oscillations with the large frequency Ω, so that we may look for the rapidly oscillating component of the solution of Eqs. (1) and (2) in the form of the Fourier series

$$\xi = \epsilon[A\cos(\Omega t) + B\sin(\Omega t)] \\ + \epsilon[C\cos(2\Omega t) + D\sin(2\Omega t)] + \cdots, \tag{4}$$

where the coefficients $A, B, \ldots$ are assumed to be slowly varying on the time scale $\sim \Omega^{-1}$. Substituting the expressions (3) and (4) into Eqs. (1) and (2) and collecting the coefficients in front of the different harmonics, we obtain an infinite set of coupled nonlinear equations,

$$\ddot{X} - \alpha X + \beta X^3 + \frac{3}{2}\epsilon^2 \beta X(A^2 + B^2 + \cdots) \\ - \frac{1}{2}\alpha\epsilon^2 A = -\gamma\dot{X} + F\cos(\omega t), \tag{5}$$

$$(-\Omega^2 A + \Omega\dot{B} + \ddot{A}) - \alpha A + \gamma(\dot{A} + \Omega B) \\ + \beta(3X^2 A + \frac{3}{4}\epsilon^2 A^3 + \cdots) = \alpha X, \tag{6}$$

$$(-\Omega^2 B - \Omega\dot{A} + \ddot{B}) - \alpha B + \gamma(\dot{B} - \Omega A) \\ + \beta(3X^2 B + \frac{3}{2}\epsilon^2 A^2 B + \cdots) = \frac{1}{2}\alpha C, \tag{7}$$

$$(-4\Omega^2 C + 2\Omega \cdot D + \ddot{C}) - \alpha C + \gamma(\dot{C} + 2\Omega D) \\ + \beta(3X^2 C + \frac{3}{2}\epsilon X A^2 + \cdots) = \frac{1}{2}A, \tag{8}$$

$$(-4\Omega^2 D - 2\Omega\dot{C} + \ddot{D}) - \alpha D + \gamma(\dot{D} - 2\Omega C) \\ + \beta(3X^2 D + 3\epsilon X A B + \cdots) = \frac{1}{2}B, \tag{9}$$

and the similar equations for the coefficients in front of the higher-order harmonics. To proceed further, we note that Eqs. (6)–(9) allow an asymptotic expansion method for the functions $A, B, \ldots$. If the parameter Ω is assumed to be large, the term $-\Omega^2 A$ in Eq. (6) may be compensated only by the term αX if one assumes $A \sim \Omega^{-2}$. From Eq. (7), which has no perturbation-induced right-hand side (rhs), it simply follows that the largest term to compensate $-\Omega^2 B$ is of order of ΩA. Such a simple consideration allows us to find asymptotic expansions for the coefficients $A, B, \ldots$ in the form of series in the parameter Ω^{-1} as follows:

$$A = \frac{a_1}{\Omega^2} + \frac{a_2}{\Omega^4} + \cdots, \quad B = \frac{b_1}{\Omega^3} + \frac{b_2}{\Omega^5} + \cdots, \\ C = \frac{c_1}{\Omega^4} + \cdots, \quad D = \frac{d_1}{\Omega^5} + \cdots. \tag{10}$$

Substituting Eq. (10) into Eqs. (6)–(9) and equating the terms of the same orders in Ω, we find

$$a_1 = -\alpha X, \tag{11}$$

$$a_2 = \dot{b}_1 + \ddot{a}_1 - \alpha a_1 + 3\beta X^2 a_1 + \gamma(\dot{a}_1 + b_1), \tag{12}$$

$$b_1 = -\dot{a}_1 - \gamma a_1, \tag{13}$$

$$b_2 = -\dot{a}_2 + \ddot{b}_1 - \alpha b_1 + 3\beta X^2 b_1 + \gamma(\dot{b}_1 - a_2), \tag{14}$$

$$c_1 = -\frac{1}{8}a_1, \quad d_1 = -\frac{1}{8}b_1, \tag{15}$$

and so on. The parameter $\delta = \epsilon/\Omega$ is assumed to be up to the order $O(1)$, but all the results are valid also for the case $\delta \ll 1$. The expansions (10) allow us to find the coefficients in each order of Ω, and all the corrections are determined by *algebraic* relations. For example, a_2 is determined by Eq. (12) through b_1 which, in turn, may be found from Eq. (13) as a function of a_1, i.e., through the slowly varying part X. This statement is valid for all

coefficients of the asymptotic expansion: The coefficients are found through *algebraic* relations and not as solutions of differential equations.

Applying the expansion (10) to Eq. (5) for the slowly varying component X, we find the result

$$\ddot{X} - \alpha X + \beta X^3 - \frac{1}{2}\alpha\delta^2 \left(a_1 + \frac{a_2}{\Omega^2} + \cdots\right)$$
$$+ \frac{3}{2}\beta\delta^2 X \left(\frac{a_1^2}{\Omega^2} + \frac{2a_1 a_2 + b_1^2}{\Omega^4} + \cdots\right)$$
$$= -\gamma\dot{X} + F\cos(\omega t). \qquad (16)$$

From here it is quite obvious how to get the first, second, and subsequent orders of the approximation to determine the averaged equation.

In the first-order approximation only the term $\sim \delta^2 a_1$ contributes, so that Eq. (16) yields

$$\ddot{X} - \tilde{\alpha}X + \beta X^3 = -\gamma\dot{X} + F\cos(\omega t), \qquad (17)$$

where

$$\tilde{\alpha} = \alpha \left(1 - \frac{1}{2}\alpha\delta^2\right). \qquad (18)$$

Equations (17) and (18) take into account an effective contribution of the rapidly varying parametric force to the "average" nonlinear dynamics and this contribution might become large when $\delta = O(1)$. Thus, the dynamics of the Duffing oscillator with a rapidly varying parametric forcing may be described by a *renormalized* Duffing equation (17) and the corrections to it are of the order ϵ^2/Ω^4. In fact, applying our expansions to get the corrections of the next order approximation, we can show that this result is still valid up to the terms of order $\mathcal{O}(\Omega^{-2})$, and the corresponding coefficients of the Duffing equation are renormalized to be

$$\tilde{\alpha} = \alpha \left[1 - \frac{1}{2}\alpha\delta^2 + \frac{\alpha\delta^2}{2\Omega^2}(\alpha + \gamma^2)\right], \qquad (19)$$

$$\tilde{\beta} = \beta \left(1 + \frac{3\alpha^2\delta^2}{\Omega^2}\right), \qquad (20)$$

$$\tilde{\gamma} = \gamma \left(1 - \frac{\alpha^2\delta^2}{2\Omega^2}\right), \qquad (21)$$

where the coefficients $\tilde{\alpha}$, $\tilde{\beta}$, and $\tilde{\gamma}$ have the same meaning as those in the standard Duffing equation (1) with $\epsilon = 0$.

III. SUPPRESSION OF CHAOS

As we have shown in the preceding section, the averaged dynamics of the Duffing oscillator subjected to the rapidly varying parametric perturbations may be described by the Duffing equation again but with *renormalized* parameters. This result is rather nontrivial and it simply means that we may apply all the results known for that equation to analyze the suppression of chaos. In particular, the threshold of chaos, which is defined by the value of the ac driving force F producing the appearance of a strange attractor in the Poincaré sections, may be

obtained by means of the classical Melnikov method [14] (see also [15, 23, 24]). The method consists of evaluating the distance $\Delta(t_0)$ between stable and unstable manifolds which, in this case, form a homoclinic loop. In fact, in the presence of dissipation the homoclinic loop is destroyed, but it may be recovered by adding a force, provided that the force amplitude exceeds a certain critical value. To find the critical value, one should check if the function $\Delta(t_0)$ changes its sign for some t_0.

The Melnikov function $\Delta(t_0)$ is defined by the relation

$$\Delta(t_0) = \int_{-\infty}^{\infty} dt \, \dot{\phi}_0(t) \mathcal{R}(\phi_0(t), \dot{\phi}_0(t), t + t_0), \qquad (22)$$

where $\phi_0(t)$ is the homoclinic orbit evaluated at the absence of perturbations (i.e., without losses and force), and $\mathcal{R}$ is the rhs of Eq. (17). We should note, however, that the Melnikov method actually deals with the occurrence of transversal homoclinic points, but it does not characterize the global dynamics of the system, so that, in general, the actual threshold observed in practice becomes "visible" a little bit above the Melnikov criterion (see, e.g., [24] for more discussions of that point).

For the Duffing oscillator, the Melnikov function is (see, e.g., Ref. [15])

$$\Delta(t_0) = \pi\omega F \sqrt{\frac{2}{\beta}}\operatorname{sech}\left(\frac{\pi\omega}{2\sqrt{\tilde{\alpha}}}\right)\sin(\omega t_0) + \frac{4\gamma\tilde{\alpha}^{3/2}}{3\beta}, \qquad (23)$$

and the condition to prevent $\Delta(t_0)$ from changing the sign is

$$\gamma > \frac{3\pi F\sqrt{\beta}\omega}{(2\tilde{\alpha})^{3/2}}\operatorname{sech}\left(\frac{\pi\omega}{2\sqrt{\tilde{\alpha}}}\right). \qquad (24)$$

The main conclusion which follows from Eq. (24) is *exponential* dependence of the rhs on $\tilde{\alpha}$. This means that if $\tilde{\alpha}$ becomes smaller, then the condition (24) may be easily fulfilled by a not very large change in α, and suppression of chaos should be observed.

Crossing of stable and unstable manifolds, as is determined by the Melnikov function, gives only the criterion for the onset of chaotic motion in the limit of low dissipation when the transient times are much longer than the time scale of the system dynamics. The critical values of the parameters allowing a chaotic motion are roughly given by the criterion that the *averaged* double-well potential changes to a single-well potential (see, e.g., [25] and discussions therein). This condition yields the critical dependence $\epsilon \approx \sqrt{2}\Omega$ which separates chaotic and regular motion for the averaged dynamics.

To support the idea formuated above, we have performed computer simulations of the system described by Eqs. (1) and (2). The differential equation was integrated by the Runge-Kutta-Fehlberg (4) and (5) method with stepsize control [26]. We extracted the Lyapunov characteristic exponents using a time-discrete decomposition method with Householder orthonormalization as proposed in [27, 28]. To increase the performance of the method, we modified the algorithm in order to simulate the continuous character of the investigated system by using the exponential $\exp(\bar{M})$ of the calculated Jacobian $\bar{M}$ rather than the linearized form $(1 + \bar{M})$. This allows

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us to increase the time steps between matrix calculations by about 3–5 without any loss of precision and time of convergence. (As is well known for the Duffing oscillator, there is only one relevant Lyapunov exponent λ. The second exponent is due to the excitation and it always equals zero, while the third one is determined by dissipation: $\gamma = -\sum_i \lambda_i$.)

The relevant Lyapunov exponent versus ϵ is shown in Figs. 1 and 2. The important conclusion which follows from such dependences is twofold. First, the Lyapunov exponent vanishes for large values of ϵ and thus a regular motion is actually recovered. Second, a set of the so-called windows where the Lyapunov exponent is sufficiently suppressed or even becomes negative is observed in the simulation. We have checked such a fact for other values of the system parameters. Such windows also depend on the frequency of the parametric force. This phenomenon can clearly be seen in Fig. 3 where we have presented different types of the system dynamics defined by the Lyapunov dimension for the same parameters as in Fig. 1. The approximate dependence $\epsilon \sim \Omega$ for the threshold of chaos is reproduced rather well by the simulations.

To display *how* the oscillations become regular, we show in Figs. 1(b)–1(f) and 2(b)–2(f) the temporal evo-

FIG. 2. The same as in Fig. 1 but for $F = 0.30$. In contrast to Fig. 1 the corresponding attractor is asymmetric for $\epsilon < 2.5$ [see (b) and (c)]. Again, a set of windows of the chaos suppression is observed.

lution of the coordinate $x(t)$ described by the Duffing oscillator and its power spectrum. Those shown in Figs. 1(b),(c) and 2(b),(c) are the system oscillations at $\epsilon = 0$ which are regularized, e.g., for $\epsilon = 15$ as is depicted in Figs. 1(e) and 2(e), respectively. We have also performed calculations in the regime of higher Lyapunov exponents, e.g., for $\gamma = 0.2$, which did not show principal differences in comparison with the case $\gamma = 0.4$ presented above. So we can say that the characteristic features observed for this kind of dynamics, namely, the suppression of chaos by a HF parametric force for large ϵ and also within certain amplitude-frequency windows, is rather common for different parameter sets.

However, we should note that at very large values of ϵ the regularized averaged dynamics may become *chaotic again*. Indeed, as follows from our Eq. (19), which determines the most critical parameter for the threshold of chaos, the higher-order correction acts with the sign $+$, so that it may help to recover the chaotic dynamics again due to the driving effect of the HF parametric modulation itself. As we have checked numerically, this occurs at $\Omega = 1.0$ for $\epsilon > 0.9$, at $\Omega = 3$ for $\epsilon > 13$, at $\Omega = 5.0$ for $\epsilon > 33$, and so on. Nevertheless, the windows where the chaos is suppressed or completely eliminated are wide enough to be of practical importance.

Finally we would like to emphasize that our numerical simulations show rather good agreement with the analysis presented even for not very large values of Ω (see, e.g., Fig. 3). At the same time we have also checked the resonant case $\Omega = 1.0$ for which, as shown in Ref. [7],
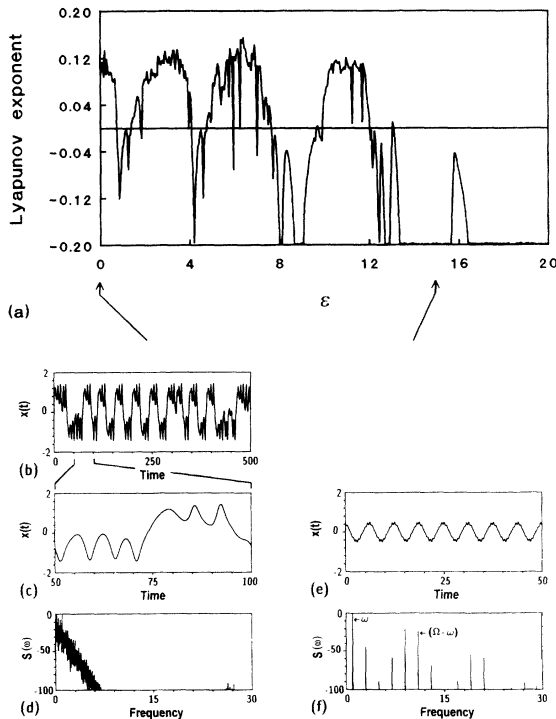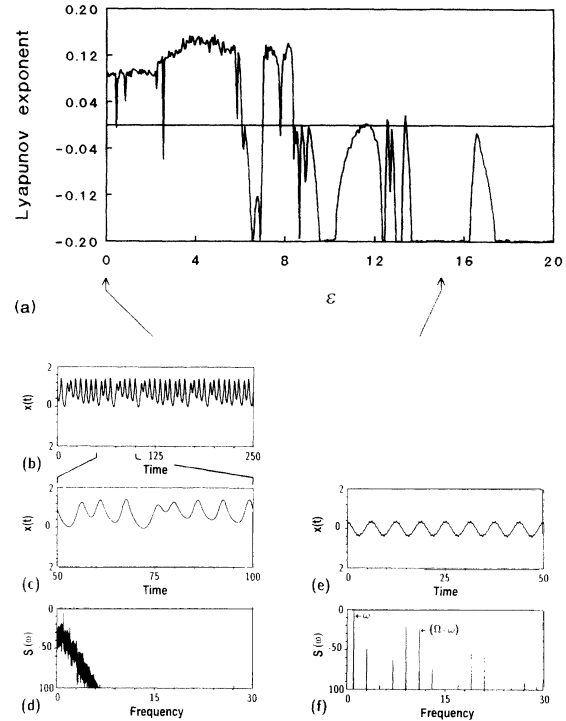

FIG. 1. Numerical simulation of Eq. (1) for $F = 0.35$, $\gamma = 0.4$, and $\omega = \alpha = \beta = 1.0$. (a) Lyapunov exponent vs amplitude of the parametric force ϵ. The negative sign of the exponent for $\epsilon > 10\sqrt{2}$ indicates suppression of chaos. Note the occurrence of periodic "windows" at lower ϵ. (b) and (c) Time dependence for the directly forced oscillator ($\epsilon = 0$) and (e) for the stabilized case ($\epsilon = 15$). (d) and (f) Corresponding power spectra. Time, frequency, and Lyapunov exponents are measured in terms of a system-time unit defined by unity.

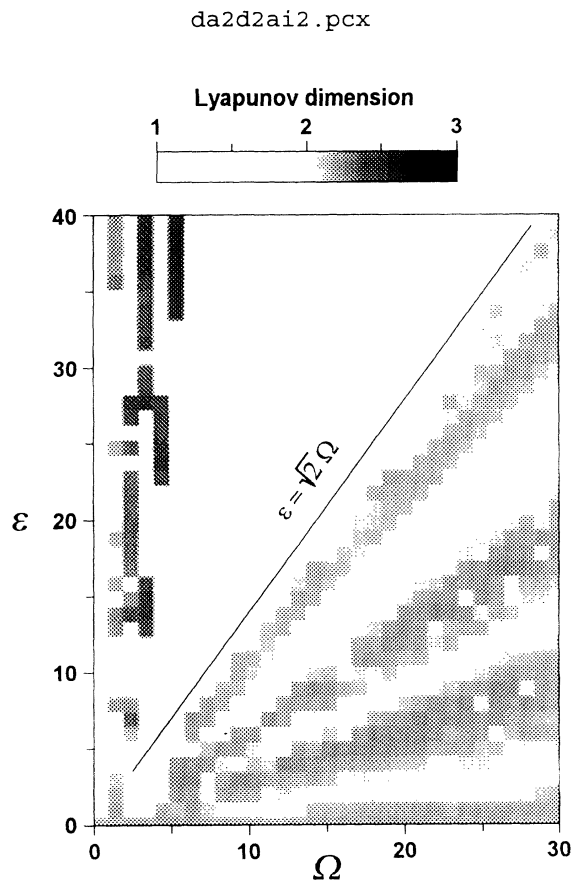da2d2ai2.pcx

Lyapunov dimension

FIG. 3. Different types of the system dynamics at the same parameters as in Fig. 1. The regions are chosen in a way that white corresponds to a Lyapunov dimension of 1, but all chaotic regimes (with dimension larger than 2) are shown in halftone. The solid line denotes the value of $\epsilon = \sqrt{2}\Omega$ above which suppression of chaos is expected. The chaotic behavior at low Ω is explained in text.

much smaller values of ϵ (e.g., $\epsilon \approx 0.1$) are necessary to suppress chaos.

IV. CONCLUSIONS

In conclusion, we have shown both analytically and numerically that suppression of chaos may be efficiently achieved by applying a periodic nonresonant paramet-

ric HF force to a chaotic dynamical system. Such a HF parametric force may decrease or even completely eliminate chaos. Taking the Duffing oscillator as a concrete but rather general example, we have elaborated an analytical approach to show how such a nonresonant suppression of chaos may be understood in the framework of the effective nonlinear dynamics averaged in the rapidly varying oscillations. In spite of the fact that the effect of the HF nonresonant parametric force looks like a shift of the system parameters, very often such a shift cannot be achieved by simply changing those (a priori fixed) parameters, and in this way one can control the averaged parameters externally. As follows from our numerical simulations, the nonresonant suppression of chaos may be observed not only at large amplitudes of the parametric force but also at smaller amplitudes showing supression of the leading Lyapunov exponent within certain amplitude-frequency windows.

As a matter of fact, this method does not require any real-time measurement of the system dynamics nor any real-time calculations, and this, as we hope, will allow us to control fast systems, e.g., faster ones than those accessible for the Ott-Grebogi-Yorke method [1]. In practice, no knowledge about the system is required at all. Thus, we expect that the approach we have used as well as our main conclusion that chaotic dynamical regimes may be sufficiently suppressed or even completely eliminated by nonresonant HF parametric perturbations are general enough to be applied to many other nonlinear models of different physical nature. We already could successfully apply this method to our spin-wave instability experiments (see, e.g., Ref. [5]) and the results will be published elsewhere.

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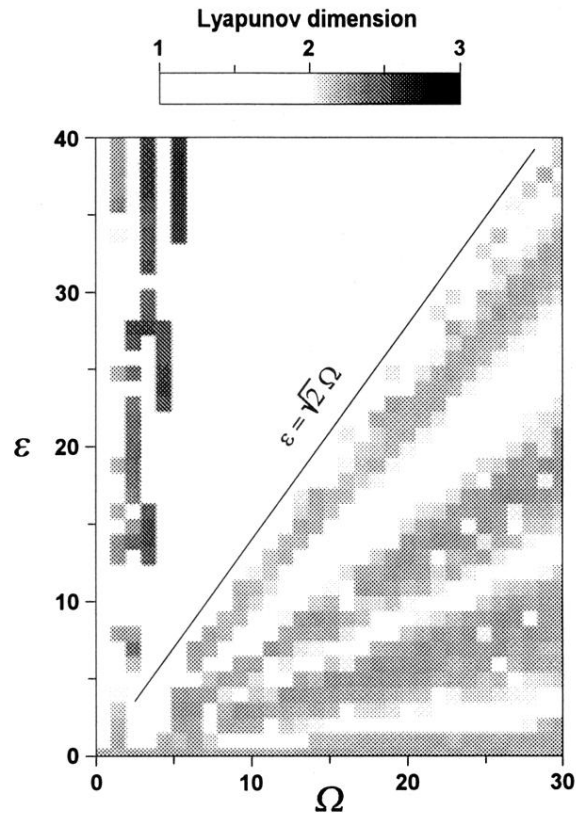[28] K. Geist, U. Parlitz, and W. Lauterborn, Prog. Theor. Phys. **83**, 875 (1990).

FIG. 3. Different types of the system dynamics at the same parameters as in Fig. 1. The regions are chosen in a way that white corresponds to a Lyapunov dimension of 1, but all chaotic regimes (with dimension larger than 2) are shown in halftone. The solid line denotes the value of $\epsilon = \sqrt{2}\Omega$ above which suppression of chaos is expected. The chaotic behavior at low Ω is explained in text.